2018 Introduction to CFD Midterm Exam

Tuesday April 24th, 2018
16:30 to 18:30

NO NOTES OR BOOKS; USE INTRODUCTION TO CFD TABLES THAT WERE DISTRIBUTED; ANSWER ALL 4 QUESTIONS; ALL QUESTIONS HAVE EQUAL VALUE.

Question #1

Starting from Newton's law $\vec{F}_y = m\frac{dv}{dt}$ and the mass conservation equation show that the y-component of the momentum transport equation for a fluid corresponds to:

$$\frac{\partial \rho v}{\partial t} + \frac{\partial \rho u v}{\partial x} + \frac{\partial \rho v^2}{\partial y} + \frac{\partial \rho w v}{\partial z} = -\frac{\partial P}{\partial y}$$

with P the pressure.

Question #2

Starting from the Euler equations

$$\frac{\partial U}{\partial t} + \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} = 0$$

and the metrics η_x, ξ_y, Ω, etc outlined in the tables, show that the Euler equations can be written in generalized coordinates in strong conservative form as follows:

$$\frac{\partial Q}{\partial \tau} + \frac{\partial G_\xi}{\partial \xi} + \frac{\partial G_\eta}{\partial \eta} = 0$$

with

$$Q \equiv \Omega \Gamma U$$

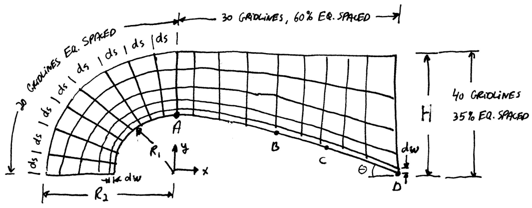
$$G_\xi \equiv \Omega(\xi_x F_x + \xi_y F_y)$$

$$G_\eta \equiv \Omega(\eta_x F_x + \eta_y F_y)$$

Outline clearly your assumptions.

Question #3

Create a grid for the following problem



given the dimensions $dw = 10^{-4}$ m, $R_1 = 0.5$ m, $R_2 = 1$ m, $x_A = 0$, $y_A = R_1$, $x_B = 0.5$ m, $y_B = 0.9R_1$, $x_C = 1$ m, $y_C = 0.7R_1$, $x_D = 2$ m, $y_D = 0$, $H = R_2$, and $\theta = 20^\circ$. Notes:

(a) Outline clearly the strategy used.

(b) Make sure that the grid spacing does not vary abruptly at any location.

(c) Points A, B, C, D are not in a straight line and should be joined using a smooth curve.

Question #4

Consider the following system of equations:

$$\frac{\partial U}{\partial t} + \frac{\partial F}{\partial x} = 0$$

with

$$U = \begin{bmatrix} \rho_1 \\ \rho_2 \\ \rho u \\ \rho E \end{bmatrix} = \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix} \quad \text{and} \quad F = \begin{bmatrix} \rho_1 u \\ \rho_2 u \\ \rho u^2 + P \\ \rho u H \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix}$$

with

$$E = \frac{\rho_1}{\rho} e_1 + \frac{\rho_2}{\rho} e_2 + \frac{u^2}{2}$$

$$P = (\rho_1 R_1 + \rho_2 R_2) T$$

$$\rho = \rho_1 + \rho_2$$

$$H = E + \frac{P}{\rho}$$

$$e_1 = \xi_1 + \xi_2 T + \xi_3 T^2$$

$$e_2 = \xi_4 T$$

and with ξ_1, ξ_2, ξ_3, ξ_4, R_1, R_2 some constants. Find $\partial F_3 / \partial U_4$ within the flux Jacobian.